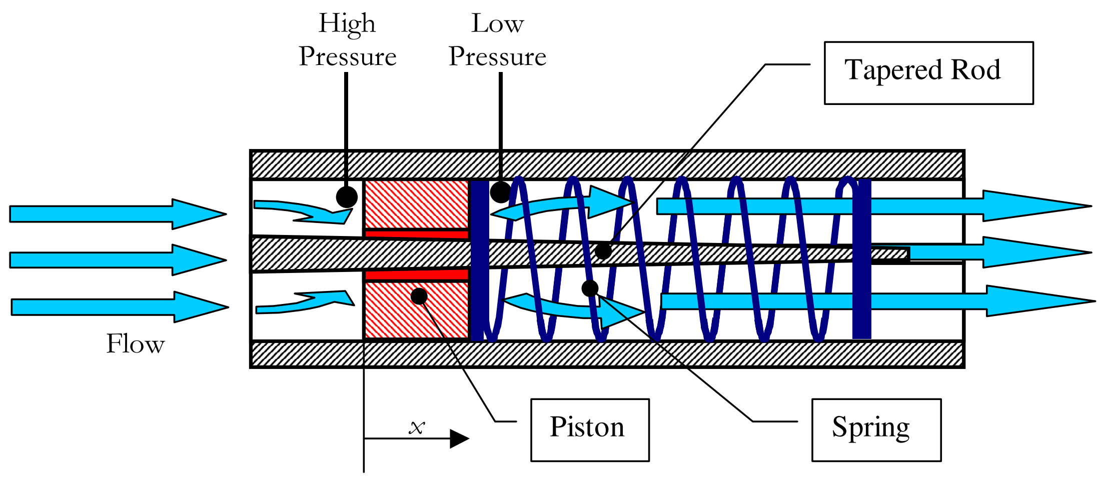
Robustica Case Study: Spring Design

Background

A local spring manufacturer was approached by an instrumentation company that was looking to take advantage of a recent increase in demand for piston flow meters. While demand had increased, the current levels of accuracy were below what the market was looking for

Such flow meters rely on a change in pressure as fluid flows through a variable area orifice formed by a hole in the piston and a tapered rod through that hole. An increase in flow will cause a resultant force on the piston, and a deflection x in a spring. As the piston moves, the orifice area becomes larger. This in turn decreases the pressure differential and the resultant force on the piston. The spring deflects further until the spring force balances the fluid force on the piston. The deflection at equilibrium is used to determine the flow, see below.



Piston flow meter operation

Typically, such flow meters provide an accuracy of about ± 5%. If this accuracy could be improved while maintaining cost competitiveness, a greater share of the expanding market would be captured. As a part of a project to improve the accuracy, the variability of the spring rate was considered. The instrumentation company asked that the quality of the springs be improved without an increase in cost.

The Problem

The spring manufacturer needed to improve the quality of their springs. However, the technology to manufacture springs was well established and optimised. The machinery used to produce the springs was top of the line and well maintained. In fact, the major sources of variability were external to the company and evident in the steel wire from which the springs were made. The two major characteristics of the wire that displayed the most variability were the diameter and the yield strength. The diameter of the wire itself has a significant effect upon the stiffness of a spring, and the yield strength influences the diameter of the spring and the number of active coils produced during manufacture.

Because the wire was bought from a supplier who themselves purchased from other various suppliers, it was not possible to implement any further controls on the production of the wire. The spring manufacturer needed to improve the quality of their springs despite having no control of the operations that contributed most significantly to variability.

The solution

Because there were no more opportunities to improve the manufacture of the springs, the actual design of the springs had to be made insensitive to the variability in the wire diameter and the material properties of the wire. Essentially, the design of the springs needed to be robustified.

The solution method

Once it was decided that robustification of the design of the springs was the solution, a method to do this was developed. The basic steps of the method were as follows:

1. Quantify the variability of the design variables that demonstrate random variability.
2. Ascertain the target stiffness of the spring and identify all failure modes. This includes determining the costs associated with each failure mode.
3. Create a mathematical model that predicts both the stiffness of a spring and the occurrence of the failure modes identified in step 2. This model must include the variables that demonstrate random variability as input variables.

4. Enter the above model into Excel and prepare the spreadsheet for robustification with Robustica.
5. Use Robustica to optimize the design so that the stiffness exhibits minimum variability and the likelihood of each failure mode is at a minimum.
6. Try the optimized design and make any necessary final changes. Ideally, these changes will only be necessary to ensure that the spring stiffness is on target and will have little effect upon the robustness of the design.

Step 1: Quantification of random variability

The yield strength of the spring steel wire is assumed to have a standard deviation 0.07 times the mean. This is true for most metals[1]. The published value for yield strength σ_y is the expected minimum value, approximately 3 standard deviations below the mean. Therefore, the mean and the standard deviation of the yield strength need to be estimated from the published value. According to the theory of maximum entropy, when only the mean and standard deviation are known the least biased distribution is a Normal distribution. Therefore, a Normal distribution will be used for the yield strength.

The diameter of the wire *d* is determined by the drawing operation used to produce the wire. The variability of the drawing operation is affected most by the wear. The drawing die is allowed to wear until the diameter of the wire is out of tolerance. The typical tolerance for wire diameter is ±0.025mm. Wear is a linear phenomenon; therefore, a Uniform distribution is assumed for the wire diameter. This means that the standard deviation is $12^{-0.5} \times R$. Where R is the total tolerance range.

Step 2: Target stiffness and failure modes

For the most common flow meter, the required spring rate is 52 $N.mm^{-1}$ and the maximum operating compression (at maximum flow) is 70mm. The spring will fail if there is any yielding at this compression. For assembly, the spring can be no longer than 150 mm. Shorter springs can be accommodated, but they must be able to accommodate the maximum operating deflection.

---

[1] **Haugen 1980.** ‘Probabilistic Mechanical design’ John Wiley and Sons

Step 3: Modeling of a spring

The stiffness of a spring k is determined by the formula below [2].

$$k = \frac{Gd}{8C^3 N_a \left(1 + \frac{0.5}{C^2}\right)}$$

Where:
G is the modulus of rigidity of the spring material
C is the spring index $\frac{D}{d}$
D is the diameter of the spring helix
N_a is the number of active coils

If the maximum shear stress is greater than the shear yield stress then the safety factor SF is below 1, and the spring will fail. The condition for the failure is expressed mathematically below.

$$SF = \frac{\tau_y}{\tau_w} \leq 1$$

Where:
τ_y is the shear yield strength
τ_w is the maximum shear stress that the spring will be subject to

The maximum shear stress that the spring will be subject to τ_w can be found by using the formula below.

$$\tau_w = \frac{8\, DK_w P}{\pi d^3}$$

Where:
K_w is the Wahl curvature correction factor $K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$
P is the maximum load applied to the spring $k \times \delta$
δ is the maximum deflection of the spring

By applying von Mises' Yield Criterion, the shear yield strength τ_y can be defined in terms of the yield strength as shown below.

$$\tau_y = \frac{\sigma_y}{\sqrt{3}}$$

[2] **Kalpakjian 1991**. 'Manufacturing Processes for Engineering Materials' Addison-Wesley Publishing Company; U.S.A.

The actual diameter of the spring is not stipulated, it must be calculated. During manufacture, the spring is initially wound to the bend diameter D_B. After this winding the coil 'springs back' to the final diameter D. An empirical relationship between D and D_B was sourced[3] and it is given below.

$$\frac{D_B}{D} = 0.5\left(\frac{D_B \sigma_y}{Ed}\right)^3 - 1.5\left(\frac{D_B \sigma_y}{Ed}\right) + 1$$

Where:
E is Young's modulus of elasticity

This formula for springback is an empirical one and therefore may have practical limitations. Further, it was originally developed for rectangular sections; however, its suitability for circular a cross section was verified. The above formula was also used to determine the bend diameter given the specified nominal diameter D_n.

The number of active coils N_a is also determined by the springback. If N_a is measured in radians then it is equal to the length of wire L_w divided by the final diameter. The length of wire is equal to the nominal diameter multiplied by the nominal number of windings N_n in radians. The result is that N_a equals N_n by the ratio of D_n to D.

$$L_w = D_n \left(N_n \times 2\pi\right)$$
$$N_a \; 2\pi = \frac{L_w}{D}$$
$$\Rightarrow Na = \frac{D_n \; N_n}{D}$$

As mentioned in step 1 the published (or nominal) value for the yield strength is three standard deviations below the mean. Further, the standard deviation is approximately 0.07 times the mean. Using this information, the following expressions for the mean and the standard deviation of the yield strength can be developed.

The mean

$$\frac{\sigma_y}{1 - 3 \times 0.07} = \frac{\sigma_y}{0.79}$$

---

[3] **Kalpakjian 1991**. 'Manufacturing Processes for Engineering Materials' Addison-Wesley Publishing Company; U.S.A.

The standard deviation

$$\frac{0.07\,\sigma_y}{1-3\times 0.07} = 0.089\,\sigma_y$$

The customer specified the maximum allowable value for the free length of the spring L_{free}. If the solid length plus the required compression is greater than this, then the redundant length L_r is less than zero, and the design has failed. The failure mode is expressed mathematically below.

$$L_r = L_{\text{free}} - \delta - L_{\text{solid}} \leq 0$$

Where:
L_{solid} is the solid length of the spring $d(N_a + 2)$

Step 5: Entering the model into Excel

With the various characteristics of the spring expressed mathematically by the above formulae, it is possible to model these characteristics in Excel. This section outlines how this was done and refers to the cells of 'Spring-example.xls', which should be downloaded with the case.

First, the constraints placed on the design and the nominal values of the design variables were specified. Cells B4 to K11

Second, the manufacture of the spring was modeled. Specifically, the springback was modeled. The appropriate bend diameter, based on the nominal values and the springback model, was found using a macro 'Springback' that ran Goalseek to find the bend diameter that would provide a springback diameter D_s equal to the nominal spring diameter D_n. This bend diameter would be used later to predict the actual spring diameter D. Cells B14 to L16

Third, the mean and the standard deviation of each random variable, yield strength σ_y and wire diameter d, were specified. Cells B20 to F21.

Fourth, intermediate variables such as the actual spring diameter and the stress in the spring were found. These variables used the random variables instead of the respective nominal variables. Cells B25 to L29.

Fifth, the characteristics of interest, the spring constant k, the safety factor SF and the interference I were calculated. Cells B33 to H35.

Step 6: The robustification

The current design for the spring is summarized in the table below. This design was used to provide the initial nominal values.

Name	Symbol	Units	Value
Wire diameter	d_n	m	0.00072
Spring diameter	D_n	m	0.016
Number of active coils	N_n	1	9.75
Free length	L_{free}	m	0.15

To have Robustica optimize the design for robustness, the cost of poor quality must be specified. It was estimated that the cost of a return (including customer dissatisfaction and damage to reputation) would be approximately $50. It is assumed that a return would occur if the spring yielded or if the spring could not compress 70mm without interference. Therefore, the cost associated with these two 'hard' failure modes is $50. It was also estimated that a spring rate deviation of 6 Nm^{-1} would compel a customer to return the product. By assuming the geometric relationship between deviation from target and poor quality cost, the quality loss coefficient for the spring k_s was found to be:

$$\$50 = k_s \left(6 \text{ Nm}^{-1}\right)^2 = k_s \; 36\left(\text{Nmm}^{-1}\right)^2$$

$$k_s = \frac{50}{36} \frac{\$}{\left(\text{Nmm}^{-1}\right)^2} = 1.39 \frac{\$}{\left(\text{Nmm}^{-1}\right)^2}$$

The information for the inputs and the outputs has now been ascertained. However, for robustification, one must specify the possible range for the adjustable variables. In this case, the design variables shown in the table above were all adjustable. The variables and their allowable range are shown in the table below. The ranges are based on restriction from the customer and suppliers.

Name	Symbol	Units	Min	Min
Wire diameter	d_n	m	0.0004	0.0015
Spring diameter	D_n	m	0.01	0.03
Number of active coils	N_n	1	1	15
Free length	L_{free}	m	0.07	0.15

With the above information specified, it became possible to complete the Excel spreadsheet for the spring, and predict the quality and robustify the design. For the original design, there was a negligible chance of the spring interfering with itself or yielding. Further, the mean spring rate was just below the target at 51.8 Nm^{-1} and the standard deviation was 2.64 Nm^{-1}.

The design was robustified two times with various settings. The first used low crossover and mutation rates whereas the second used higher rates to search for any remaining improvements not yet tried. The first robustification was responsible for the major improvement and the second robustification, which included the optimum design values from the first robustification, provided further reasonable improvements. The most robust design found is summarized in the table below.

Name	Symbol	Units	Value
Wire diameter	d_n	m	0.001169815
Spring diameter	D_n	m	0.026448611
Number of active coils	N_n	1	15
Free length	L_{free}	m	0.15

After robustification, the mean spring rate had dropped to 51.7 Nm^{-1} and the standard deviation had been reduced to 1.89 Nm^{-1}. While there had been a significant reduction in the random variability of the spring rate, which would improve the quality of the flow meters, the chance of the spring interfering with itself or yielding had remained negligible. Therefore, the quality of the spring and the flow meter had been increased significantly without the need for significant changes to the manufacturing operation.

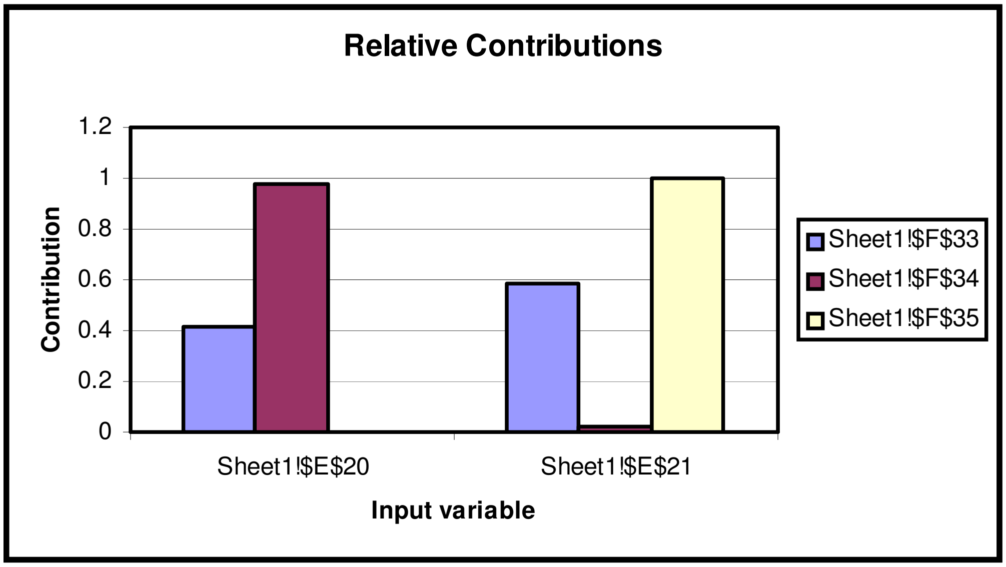
Step 7: Final adjustments

Before the new design was tried physically, two actions were taken. First, the values were adjusted so that they were more rounded. Second, a Monte Carlo simulation was run to ensure the accuracy of the optimization. The results were in agreement with the Monte Carlo simulation, and the design was then tried physically to ascertain any further adjustments to the design that might be needed.

Further gains

It is possible that the quality of the spring will need to be improved further. As already mentioned, there is little control over the two major sources of random variability: the wire diameter and the yield strength. Nevertheless, if a further improvement is required, these variables will need to be better controlled. In preparation for this eventuality, the spring company thought it wise to consider possible action that could be taken. Each action considered will attempt to control the diameter, the yield strength or both. To ascertain the potential benefit of a proposed action, it is valuable to know which of the random variables contributes most to the variability of the spring rate. Therefore, the contribution chart shown in the figure below was produced. The cell of interest is 'F33' (this holds the value for the spring rate) and is represented by the light blue series.

It can be seen that the variable allocated to cell 'E21' (wire diameter) contributes about 60% of the variability in the spring rate. Therefore, actions that focus on controlling the wire diameter (such as bringing the drawing operation in house so that it will be better controlled) will offer greater benefit. This knowledge allows the company to focus on actions that are more likely to be of value: those that control the wire diameter.



Contribution chart post robustification

Summary

The above has shown how one can model not only the product (in this case a spring) but also the manufacturing process of the product (in this case, springback needed to be considered). Further, it has shown the importance of the user being aware of the source of random variability and how that variability propagates. An alternative approach to modeling the system would have been to allocate assumed or 'typical' standard deviations to each of the input variables, ignoring the actual influence of the yield strength and the manufacturing process. Such an approach would have produced different results, which would have been erroneous.

The case also demonstrated that the random variables could at times be different to the variables that are adjusted. For this case, the design of the spring was modified to reduce sensitivity to the random variability in the yield strength and the wire diameter. This further demonstrates the importance of the user understanding the flow of random variability through a system when constructing the system model.

Finally, the case shows that even after robustification has been completed, you should always be mindful of what can be done to improve the system further. It is possible to ascertain the major contributors to the variability through the contribution chart. With this knowledge, efforts are focused on those input variables that offer the greatest potential.