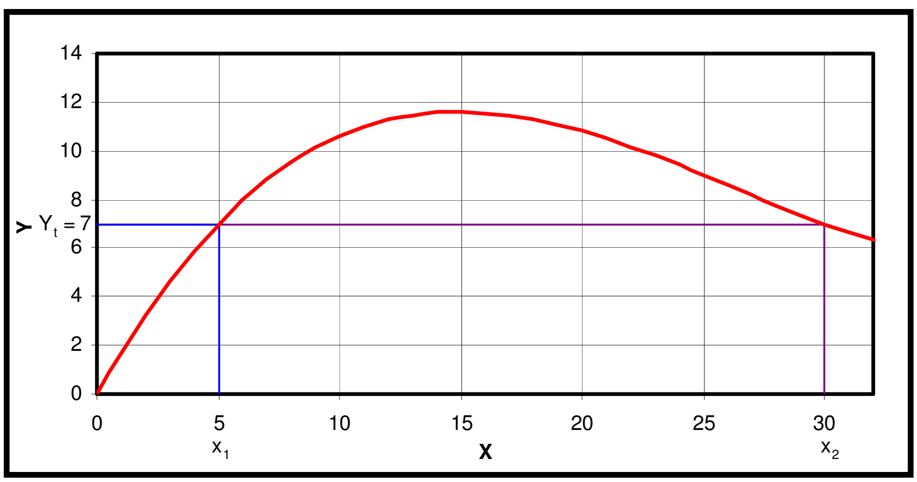
Robustification

Introduction

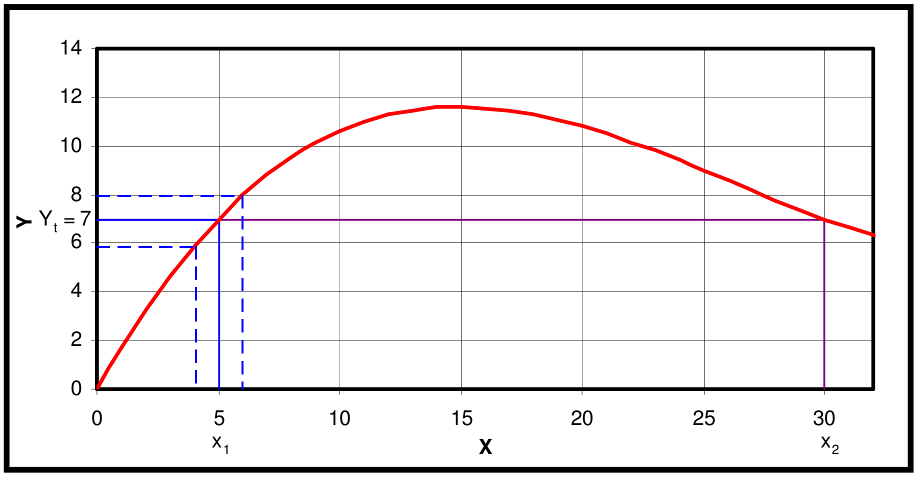
Robustification allows us to make systems less sensitive to various sources of random variability. This is done by adjusting the nominal values of the design variables; not by restricting the actual random variability. Because this reduces the need to install more costly manufacturing equipment or put extra effort into the tight control of various operations, it is a more cost effective approach. Many not familiar with robustification find this difficult to believe (if something sounds too good to be true…). Therefore, this article will explain the principles of robustification in a graphical, and easier to understand, method.

Principles of robustification

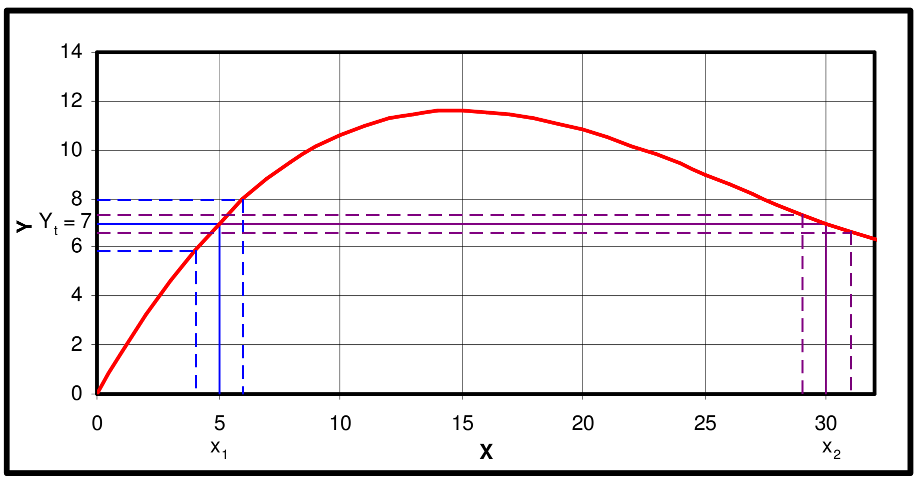
To demonstrate the principles of robustification, consider an arbitrary system that can be modelled with the plot below. The desired value for the output of the system is $y = 7$. For this particular system we have two choices for x that will provide this output: 5 and 30.



If we apply the traditional approach we would not be likely to ascertain any difference between one x value and the other. However, if we consider the minimum and maximum values that x can take, we start to get a different appreciation for the importance of selecting the one value for x or the other. It will be assumed that the tolerance on x is ±1. Let's first consider the variance effects that can be expected in y if x =5 is chosen. We see that y could take a value as high as 8 or slightly less than 6.



Now let's see what happens when we consider the variability that can be expected in y if we choose x =30 (and the variance in x remains the same). We can see that by choosing x =30 there will be less variance in y ($6.5 \leq y \leq 7.5$). If we consider the plot some more some more, we see that the reason for the reduction in the random variability of y is that at x =30 the gradient is smaller. This is the basis of robustification.



In reality, we will typically have a number of input variables. Therefore, instead of looking for the viable point on a curve with the smallest gradient, we look for the viable combination of input variables on the hyper-plane (the surface determined by the relationship between the input variables and the output) is at its flattest. By viable, it is meant that the output variable is equal to the target value.

Post robustification

After robustification, it might be that the random variability displayed by the output is still considered excessive. If such an event were to occur, then it would be of interest to us to determine which of the input variables should be most tightly controlled. If the slope of the relationship between a particular input variable and the output were relatively low, then there would be little benefit in reducing the random variability of that input variable. Therefore, we wish to find those input variables that have the greatest slope (or partial derivative) in their relationship with the output. The random variability of such input variables will be the most amplified, and control this randomness would provide maximum benefit. Once these input variables have been identified, action can be taken to reduce the random variability that they display.

Parameter design and robustification

Some refer to the process of robustification, as described above, as parameter design. They also consider the adjustment variables to be parameters. Instead, they include the identification of those input variables that should be tightly controlled as a part of robustification. It is not the purpose of this article to discuss the appropriate definition, but it was felt that this difference in opinion should be noted.

Summary

Robustification reduces the sensitivity of a system to random variability in the input variables and parameters. It is achieved by determining the combination of input variables that provide the desired output value and correspond to the flattest region of the system's hyper-plane. Finally, it should be noted that some say that robustification includes the determination and control of those input variables that should be more tightly controlled.